MAS114 Problems

Sheet 1 (Week 1)

Preamble

Themes from this week (ask your tutorial staff if you're stuck): 1. What is mathematics? 2. The importance of rigour. 3. Sets of numbers. 4. Sets as abstract objects.

Work on these problems one at a time in small groups (of around four). For many problems there is designed to be a lot of work that's best shared; and for others discussion is vital to understanding.

1

Work out the eight terms $a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7$ for each of the following sequences. What do we learn about the meaning of the phrase "the sequence $1, 2, 4, 8, \ldots$"?

(i) the sequence defined by $a_n = 2^n$ *(this is the "powers of two", or "doubling" sequence)*;

(ii) the sequence defined by $a_n = (n^3 + 5n + 6)/6$ *(this is called the "cake sequence", as it's the number of pieces you can cut a cake into with n cuts without rearrangement)*;

(iii) the sequence defined by setting $a_{-3} = a_{-2} = 0$, $a_{-1} = a_0 = 1$, and $a_n = a_{n-1} + a_{n-2} + a_{n-3} + a_{n-4}$ for $n \geq 1$;

(iv) the sequence defined by setting $a_n = (n^4 - 2n^3 + 11n^2 + 14n + 24)/24$;

(v) the sequence defined by taking a_n to be the sum of the digits of 2^n;

(vi) the sequence defined by setting a_0 to be 1, and a_n to be 1 plus the sum of all the *individual digits* of all previous terms of the sequence (not the terms themselves, just their digits);

(vii) the sequence defined by taking a_n to be the remainder left by dividing 2^n by 15.

This is quite a lot of work, so divide it up between you.

2

Alice plays a game with Bob. Alice starts by saying either "1", or "1, 2", or "1, 2, 3". Whatever she says, Bob says either the next one, two, or three natural numbers. Then Alice says the next one, two or three natural numbers, and so on.
For example, Alice might say "1, 2", and then Bob might say "3, 4, 5", and then Alice might say "6", and then Bob might say "7, 8".
The *loser* is the player who says "21". If both players play perfectly, which player should win: Alice or Bob?

For discussion: Think about what this question even means: what is a winning strategy for a game?

3

What is the largest number of each of the following types of chess pieces that could be put on an empty 8×8 chessboard so that no piece attacks any other:

(a) rooks?
(b) kings?
(c) bishops?
(d) knights?
(e) queens?

In each of the five cases, try to both find a configuration with as many pieces as possible, and prove that it's not possible to have a larger number on the board.
(If you don't know how chess pieces move, don't just sit about: ask somebody else, or use the internet to find out.)

For discussion: Is it convincing to take a board, put a lot of pieces on, and say that no more can be added to that configuration, so it's the maximum?
What would be a completely convincing argument?

Reminder

Hand in the *homework* next week! You'll find this on the course webpage.