MAS114 Problems

Sheet 3 (Week 3)

Preamble

Themes from this week (ask your tutorial staff if you're stuck): 1. Negation. 2. The contrapositive. 3. "And" and "Or". 4. Quantifiers. 5. Induction.

Work on these problems one at a time in small groups (of around four). For many problems there is designed to be a lot of work that's best shared; and for others discussion is vital to understanding.

1

Which of the following statements are true?

(i) $\forall m \in \mathbb{Z}, \quad \exists n \in \mathbb{N} \quad \text{s.t.} \quad m < n,$

(ii) $\forall m \in \mathbb{Z}, \quad \exists n \in \mathbb{N} \quad \text{s.t.} \quad m > n,$

(iii) $\exists m \in \mathbb{Z} \quad \text{s.t.} \quad \forall n \in \mathbb{N}, \quad m < n,$

(iv) $\exists m \in \mathbb{Z} \quad \text{s.t.} \quad \forall n \in \mathbb{N}, \quad m > n,$

For discussion: Can you find a good way of unravelling these statements in your head?

2

Let P and Q be statements. Complete the following truth tables, where 1 means true, and 0 means false:

P	Q	$\neg P$	$\neg Q$	$P \vee Q$	$\neg(P \vee Q)$	$(\neg P) \wedge (\neg Q)$
0	0	1		0		
0	1	1		1		
1	0	0		1		
1	1	0		1		

P	Q	$\neg P$	$\neg Q$	$P \wedge Q$	$\neg(P \wedge Q)$	$(\neg P) \vee (\neg Q)$
0	0	1		0		
0	1	1		0		
1	0	0		0		
1	1	0		1		

For discussion: What is the relationship between the right-hand columns of these two tables? And what does this *mean*? Also, what does this have to do with Question 2 in the homework you've just handed in?

3

Given two propositions P and Q, there are exactly sixteen possible truth tables: sixteen different ways of filling in the table

P	Q	result
0	0	?
0	1	?
1	0	?
1	1	?

with 0's or 1's. Some of them appeared in the question above, but others didn't. Can you write them all down, and describe how to obtain them all using P, Q, $\neg$, $\vee$ and $\wedge$ (and parentheses)? For example, $P \wedge Q$ gives the column whose values are 0,0,0,1?

For discussion: Can you describe some of them in several different ways? Is there any reason to think that some descriptions might be better than others? Is there any other way you'd like to describe them?

4

Continuing the previous question, how many possible truth tables would there be if there were three variables P, Q and R? What if there were four variables, or five?

For discussion: Is this a good way of understanding logic, as the number of variables grows large?

5

Let A be a set with four elements, and B be a set with five elements. How many injections are there from A to B? How many from B to A? How many surjections are there from A to B? How many from B to A?