

# MAS114 Problems

## Sheet 5 (Week 5)

### Preamble

Themes from this week (ask your tutorial staff if you're stuck): 1. There are infinitely many prime numbers. 2. Sieving to find primes. 3. The greatest common divisor. 4. Coprime numbers. 5. Euclid's algorithm.

Work on these problems one at a time in small groups (of around four). For many problems there is designed to be a lot of work that's best shared; and for others discussion is vital to understanding.

1

Is it true, for integers  $a, b, c$ , that if  $a \mid bc$  then either  $a \mid b$  or  $a \mid c$ ? Give either a proof or a counterexample.

2

Find three numbers  $a, b$  and  $c$  such that

- $a$  and  $b$  have 20 as a factor (and there's no larger number which is a factor of both),
- $b$  and  $c$  have 6 as a factor (and there's no larger number which is a factor of both),
- $c$  and  $a$  have 14 as a factor (and there's no larger number which is a factor of both).

3

Show that all the nine numbers

$$10! + 2, \quad 10! + 3, \quad \dots, \quad 10! + 10$$

are all composite.

Prove similarly that, for any  $n$ , there is a block of  $n$  consecutive composite integers.

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*For discussion:* Do you think this is an *efficient* solution to the problem? What is the first block of five consecutive composite numbers? Are the numbers smaller than your approach above gives you?

4

- (i) The numbers 3, 5, 7 are all prime, and are consecutive odd numbers. Does it ever happen again that the three consecutive odd numbers

$$2n + 1, 2n + 3, 2n + 5$$

are all prime?

- (ii) Can you find five primes in *arithmetic progression*, that is, numbers  $a, a + d, a + 2d, a + 3d, a + 4d$  which are all prime? Can you find six in arithmetic progression?

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*For discussion:* Given an arithmetic progression  $a, \dots, a + 5d$  of six primes, can you say anything about what  $d$  must be like? How about for an arithmetic progression of  $n$  primes?

Can you find some other configurations of primes that you guess *will* occur repeatedly? (Can you find more of them?) Can you find others that you guess *won't*?

5

Two numbers are said to be *coprime* if their greatest common divisor is 1: in other words, if they have no positive factors in common except 1.

- (i) What's the largest subset of  $\{1, \dots, 20\}$  you can find, in which *every* pair of two different integers is coprime?
- (ii) What's the largest subset of  $\{1, \dots, 20\}$  you can find, in which *no* pair of two different integers is coprime?

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*For discussion:* Can you think of any way of proving that these are the largest possible subsets, in each case?