

MAS114 Homework Problems

Week 2 (hand in in week 3)

1. Write down all the elements of each of the following sets:

(a) $\{a \in \mathbb{N} \mid a^2 < 9\}$;

(c) $\{a \in \mathbb{Z} \mid a^2 < 9\}$;

(b) $\{a \in \mathbb{N} \mid a^2 \leq 9\}$;

(d) $\{a \in \mathbb{Z} \mid a^2 \leq 9\}$.

2. Suppose U is a set. If X is a subset of U , we'll write \overline{X} for $U \setminus X$ for the duration of this question. (*This is common notation whenever we work at length with subsets of some particular set.*)

Let A and B be subsets of U . Show that:

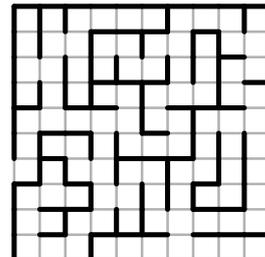
(i) $\overline{A \cup B} = \overline{A} \cap \overline{B}$

(ii) $\overline{A \cap B} = \overline{A} \cup \overline{B}$

(*These are called De Morgan's laws. Remember, the best way to prove two sets are equal is often to prove that each is contained in the other. So for each of these two you have two containments to prove: the left-hand side in the right-hand side, and vice versa.*)

3. **Challenge:** I foolishly left my dog in a maze, and I'm not allowed in to retrieve him. The maze is a $10 \text{ m} \times 10 \text{ m}$ grid, where some of the grid edges are walls and some aren't. I can't remember anything about where the walls are, but this picture shows an example of a similar maze.

My dog is very obedient, but not very clever. He understands the instructions "walk 1 m forwards", "turn 90° left", and "turn 90° right" but nothing else of any use. If I shout for him to walk forwards and that would result in him walking into a wall, he'll just do nothing for that order and wait patiently for the next one, wagging his tail.



I can't see into the maze, and have no idea whether he is able to walk forwards or not after each time I tell him to do so.

Is there a sequence of orders I could shout that would get him out of the maze, no matter what the maze looks like and no matter which square I left him at, and no matter which direction I left him facing?

[Please hand in attempts to the Challenge problem on a separate sheet of paper so they can make their way to Dr Cranch more easily.]