MAS114 Homework Problems

Week 3 (hand in in week 4)

1. An *even number* is an integer that can be written in the form $2k$ for some integer k. An *odd number* is one that can be written in the form $2k+1$ for some integer k. Using these definitions *and no other facts you may happen to know about odd or even numbers*, prove the following implications:

   (a) If n is even, then n^2 is even.

   (b) If n and m are odd, then $n+m$ is even.

   (c) If n and m are odd, then nm is odd.

   State the converse of each of the above implications. Do you think they are true or false?

2. Prove by induction that

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6},$$

   for all natural numbers n.

3. **Challenge:** Suppose that we have some positive integers (not necessarily distinct) whose sum is 100. How large can their product be? You should prove your answer is best.

   [*Please hand in attempts to the Challenge problem on a separate sheet of paper so they can make their way to Dr Cranch more easily.*]