

MAS114 Homework Problems

Week 4 (hand in in week 5)

1. Prove that for all natural numbers $n \geq 2$, we have

$$\left(1 - \frac{1}{2^2}\right) \left(1 - \frac{1}{3^2}\right) \cdots \left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n}.$$

2. Prove carefully that $4^n < n!$ for all natural numbers $n \geq 10$.

(The quantity $n!$ denotes the factorial of n : the product of the natural numbers from 1 up to n .)

3. **Challenge:** The *Fibonacci numbers* are a function $F : \mathbb{N} \rightarrow \mathbb{N}$ defined by $F(0) = 0$, $F(1) = 1$, and $F(n) = F(n-1) + F(n-2)$ for $n \geq 2$. So, for example, $F(2) = 1$, $F(3) = 2$, $F(4) = 3$, $F(5) = 5$, and so on.

Is $F(2013)$ even or odd? Find an odd prime factor of $F(2013)$.

[Please hand in attempts to the Challenge problem on a separate sheet of paper so they can make their way to Dr Cranch more easily.]