MAS114 Homework Problems

Week 5 (hand in in week 6)

1. For each of the following statements, either prove them or find a counterexample. Your proofs should proceed *directly from the definition of divisibility.*

   (i) Let a, b, c be integers. If $a \mid b$ and $b \mid c$, then $a \mid c$.

   (ii) Let a, b, c be integers. If $a \mid b$ and $a \mid c$, then $a \mid b + c$.

   (iii) Let a, b, c be integers. If $a \mid b + c$, then $a \mid b$ and $a \mid c$.

   (iv) Let a, b, c be integers. If $a \mid b$ and $a \mid c$, then $a \mid bc$.

   (v) Let a, b, c be integers. If $a \mid bc$, then $a \mid b$ and $a \mid c$.

   (vi) Let a, b, c, d be integers. If $a \mid b$ and $c \mid d$, then $ac \mid bd$.

2. **Challenge:** Show that any two numbers of the form $2^{2^n} + 1$ are coprime (where n is a nonnegative integer), and thus give an alternative proof that there are infinitely many primes.

   [*Please hand in attempts to the Challenge problem on a separate sheet of paper so they can make their way to Dr Cranch more easily.*]