

# MAS114 Homework Problems

## Week 8 (hand in in week 9)

1. Find all solutions to the following congruence equations: in each case, either state that there are no solutions, or give them in the form  $x \equiv a \pmod{b}$ .
  - (a)  $6x \equiv 10 \pmod{14}$ ;
  - (b)  $6x \equiv 9 \pmod{14}$ ;
  - (c)  $5x \equiv 8 \pmod{14}$ ;
  - (d)  $7x \equiv 8 \pmod{14}$ .

2. Find all solutions to the congruence equation

$$143x \equiv 243 \pmod{343}.$$

3. **Challenge:** In 1994, Andrew Wiles, building on work of many other people, proved *Fermat's Last Theorem*, that there are no solutions to the equation

$$a^n + b^n = c^n,$$

where  $a, b, c$  and  $n$  are positive integers and  $n > 2$ .

Show that there *are* infinitely many solutions in positive integers to

$$a^{34} + b^{34} = c^{35}.$$

Then show that there are also infinitely many solutions in positive integers to

$$a^{51} + b^{52} = c^{53}.$$

[Please hand in attempts to the Challenge problem on a separate sheet of paper so they can make their way to Dr Cranch more easily.]