

MAS114 Homework Problems

Week 9 (hand in in week 10)

1. Solve the following simultaneous congruence equations:

(a) $x \equiv 5 \pmod{7}$, $x \equiv 2 \pmod{6}$;

(b) $x \equiv 5 \pmod{8}$, $x \equiv 2 \pmod{6}$;

(c) $x \equiv 5 \pmod{9}$, $x \equiv 2 \pmod{6}$;

(d) $x \equiv 17 \pmod{41}$, $x \equiv 36 \pmod{43}$.

2. Show that $17 \mid (3^{32} - 2^{32})$ using Fermat's Little Theorem.

3. **Challenge:**

(i) Show that 561 is not prime.

(ii) Show that, even though 561 is not prime, if $\gcd(a, 561) = 1$, then $a^{560} \equiv 1 \pmod{561}$.

This shows that one possible converse of Fermat's Little Theorem is not true. Numbers with this property, of being composite but "apparently prime" from the point of view of Fermat's Little Theorem, are called Carmichael numbers.

[Please hand in attempts to the Challenge problem on a separate sheet of paper so they can make their way to Dr Cranch more easily.]