MAS114 Homework Solutions

Week 1 (hand in in week 2)

1. Learn the Greek alphabet: learn the names of all the lower-case letters

$$\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta, \theta, \iota, \kappa, \lambda, \mu, \nu, \xi, o, \pi, \rho, \sigma, \tau, \upsilon, \phi, \chi, \psi, \omega.$$

(They're among the commonest of the unfamiliar symbols that mathematicians use. If you're Greek or Cypriot, you have an advantage, but you should probably get used to the unusual way their names are pronounced in English.)

Solution You're on your own with this one (but, then, millions of primary school children in two countries have managed it, so it's not so very difficult).

2. Which of the following rules define a function? For those that are functions, are they injective? Are they surjective? Are they bijective? Give brief explanations where necessary.

(i) $f : \mathbb{N} \to \mathbb{R}$ defined by $f(n) = \sqrt{n}$;

(ii) $g : \mathbb{Z} \to \mathbb{R}$ defined by $g(n) = \sqrt{n}$;

(iii) $h : \mathbb{Z} \to \mathbb{N}$ defined by $h(n) = |n|$;

(iv) $i : \mathbb{N} \to \mathbb{N}$ defined by taking $i(n) = 100 - n$.

(v) $j : \mathbb{Z} \to \mathbb{Z}$ defined by $j(n) = -n$;

(vi) $k : \mathbb{R} \to \mathbb{Z}$ defined by taking $k(x)$ to be the closest integer to x.

Solution

(i) This is a function, as every natural number has a real square root. It's injective, because every natural has a different square root. It's not surjective, for example because $1/2$ is not the square root of a natural number. Hence it's also not bijective.

(ii) This is not a function, because -1 does not have a real square root.

(iii) This is a function: every integer has an absolute value which is a natural number. It's not injective, because $h(-1) = h(1) = 1$. But is surjective, since for every $n \in \mathbb{N}$, we have $h(n) = n$. It can't be bijective, as it isn't injective.

(iv) This is not a function: we have $i(101) = -1 \notin \mathbb{N}$.

(v) This is a function: every integer can be negated to give another integer. It's injective, since no two different integers have equal negations. It's surjective, since for any $n \in \mathbb{Z}$ we have $j(-n) = n$. Hence it's bijective too!

(vi) This is "almost" a function, but in fact it isn't. For example, there is no integer closest to $1/2$, as 0 and 1 are equally close.

3. **Challenge:** How many subsets are there of $\{1, 2, 3, \ldots, 19, 20\}$ which contain no two consecutive elements? (For example, $\{1, 4, 18\}$ is okay, but $\{1, 4, 17, 18\}$ is not okay since it contains the consecutives 17 and 18.)

[*Please hand in attempts to the Challenge problem on a separate sheet of paper so they can make their way to Dr Cranch more easily.*]