# MAS114: Lecture 7

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2018–2019

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Kevin Buzzard, Hicks Lecture Theatre C, Wednesday 2pm

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I argue that pure mathematics is walking inexorably towards a cliff edge, and that anyone who believes that current pure mathematics is rigorous, or a science, needs to wake up and look at the facts, which there will be plenty of in this talk, and they are not pretty. Are our results reproducible? Does it matter? What *is* mathematics? Can computer scientists save us? Can undergraduates save us? I hope so. This talk is about pure mathematics but will be accessible to undergraduates, mathematicians both pure and applied/applicable and computer scientists.

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Clearly this statement is complete and utter rubbish.

> If you believe that induction is a reliable method of proof (and I do, and I hope you do too), then it had better be the case that we're not using induction correctly.

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If you don't have a base case, such as $P(0)$, then it's of no use to prove that $P(k) \Rightarrow P(k+1)$ for all $k$. It's no use to be able to climb a ladder if the bottom of the ladder is unreachable.

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We'll take $P(1)$ as the base case of the induction. This is the statement "Given any one horse, all of them have the same colour": this is obviously true.

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Now we'll prove the induction step. We will assume that $P(k)$ is true ("given any $k$ horses, all of them have the same colour"): our job is to prove that $P(k+1)$ is true ("given any $(k+1)$ horses, all of them have the same colour").

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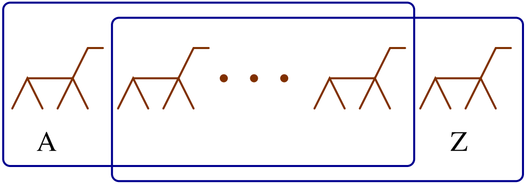

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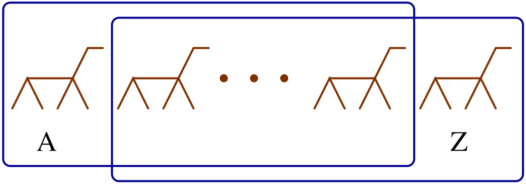


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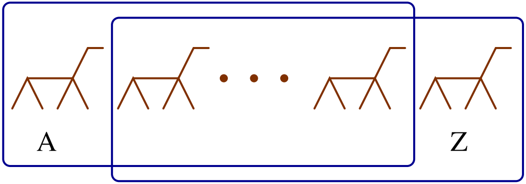


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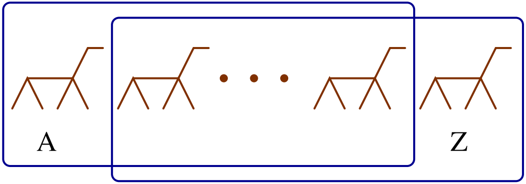


Excluding Alice, there are $k$ horses, which all have the same colour, by the induction hypothesis. So all the horses except Alice have the same colour as Zebedee.

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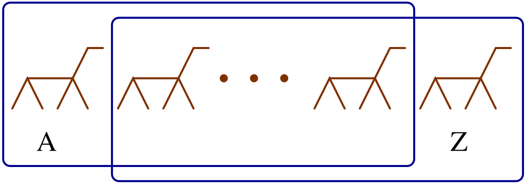


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In fact, it's a parody of a *valid* style of argument. If it is the case that *any two things are the same*, then we could prove using exactly this method that they're *all the same*. In fact, this is something you already know, since "all are alike" and "no two differ" are synonymous phrases.

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Perhaps you want to think of that as saying "if have a door which leads to the fifteenth rung of a ladder, and you know how to climb ladders, then you can get to every rung above the fifteenth".

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Actually, you should have been prepared for this variant: my induction proof that "all horses have the same colour" started with 1, not 0. (Okay, that proof was wrong. But there was nothing wrong with *that bit* of the proof: there's nothing wrong with induction starting from 1. It was something else that was wrong).

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In order to prove this, we assume $P(0), \ldots, P(k)$ are all true and have to prove that $P(0), \ldots, P(k+1)$ are all true. But then all of these except the last are assumptions: what is left is to prove $P(k+1)$ assuming $P(0), \ldots, P(k)$, and that's exactly the induction step of a strong induction.

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### Proof.

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So we proved $P(0)$, and we proved $P(0) \Rightarrow P(1)$ by proving $P(1)$, and then we proved $P(0) \wedge \cdots \wedge P(k) \Rightarrow P(k+1)$ by proving $P(k-1) \wedge P(k) \Rightarrow P(k+1)$.

# Some thoughts on that

That strong induction argument really had *two base cases* before the induction step.
So we proved $P(0)$, and we proved $P(0) \Rightarrow P(1)$ by proving $P(1)$, and then we proved $P(0) \wedge \cdots \wedge P(k) \Rightarrow P(k+1)$ by proving $P(k-1) \wedge P(k) \Rightarrow P(k+1)$.
I like to think that the proof was arranged according to the shape of the definition of the Fibonacci numbers: that definition has two base cases $F_0 = 0$ and $F_1 = 1$, and a step $F_{n+2} = F_{n+1} + F_n$. This is not a rare coincidence.