MAS114: Lecture 8

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2018–2019

A Horse of a Different Colour

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If you see a proof which you consider to be particularly well written, try to *learn from it* as an example of what's good to do. Similarly, if you see a proof which you consider to be particularly badly written, maybe you can learn something from the experience about what to avoid!

Also, *read it back* to yourself (or better yet, get a colleague to read it). You're writing it so others can read: it's good to test it to make sure this is possible. This goes particularly for proofs containing large amounts of symbols: these can be very hard to read, and reading it back to yourself is probably the best way of detecting this.

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Alternatives to symbols

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Here are some phrases which do the job of "∴":

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so,

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so, *hence*,

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so, *hence*, *therefore*,

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Here are some phrases which do the job of "∴":

so, hence, therefore, thus,

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Here are some phrases which do the job of "∴":

so, *hence*, *therefore*, *thus*, *consequently*,

Alternatives to symbols

Here are some phrases which do the job of "∴":

so, *hence*, *therefore*, *thus*, *consequently*, *as a result*,

Alternatives to symbols

Here are some phrases which do the job of "∴":

so, hence, therefore, thus, consequently, as a result, accordingly, for this reason,

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Here are some phrases which do the job of "∴":

*so, hence, therefore, thus, consequently, as a result,
accordingly, for this reason, and so,*

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Here are some phrases which do the job of "∴":

so, hence, therefore, thus, consequently, as a result, accordingly, for this reason, and so, and in particular,

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> because, since, as a result of, as we have,

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When it's really important, I like to use both to make sure the point is clear. I might write,

Let $A = \sum_{i=1}^{n} a(i)$ be the sum of the first n values of a, and let $p > A$ be a prime number greater than A.

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Many novices write this, wanting it to mean:

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However, experienced readers will read it as

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This is of course nonsense: equations don't really have squares, and $5 = 25$ is an invalid equation anyway.

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Let $P(n) = a_n = 3^n + 1$.

This has $P(n)$ as a *number*, not a *statement*. Instead write:

Let $P(n)$ be the statement that $a_n = 3^n + 1$.

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  - ‣ have a descriptive section heading.

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you some things to check: that *every* element of the domain gives
a *unique* element lying *inside* the codomain. Unless they're
obviously true, it could be that these checks are the hardest and
most interesting part of the proof. They could even be lies.

Explain the structure

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Also, some explanation at the end is important. When you reach a conclusion, why do you think that what you have written actually means you have finished the proof?

If the proof is long, then regard it as being made of several parts. Give each part an explanation when you start and when you finish it.

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> *The base case, where n = 0, is the statement "blah blah blah". But that's true, because blah blah blah blah blah.*

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> *We'll prove this by induction on n.*
> *The base case, where $n = 0$, is the statement "blah blah blah". But that's true, because blah blah blah blah blah. Now we need the induction step: we assume the statement true for $n = k$, and prove it for $n = k + 1$.*

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> *The base case, where n = 0, is the statement "blah blah blah". But that's true, because blah blah blah blah blah. Now we need the induction step: we assume the statement true for n = k, and prove it for n = k + 1. So we're assuming that blah blah blah, and have to prove that blah blah blah.*

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The base case, where n = 0, is the statement "blah blah blah". But that's true, because blah blah blah blah blah. Now we need the induction step: we assume the statement true for n = k, and prove it for n = k + 1. So we're assuming that blah blah blah, and have to prove that blah blah blah.
But blah blah blah blah blah.

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> *But blah blah blah blah blah. Also, blah blah blah blah blah. So, in conclusion, blah blah blah, which is what we had to prove. That finishes off the induction step, and so completes the proof.*